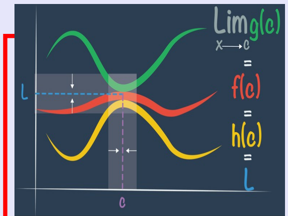
Math 261

Fall 2022

Lecture 9



for $\varepsilon>0$, find a $\delta>0$ such that

$\lim_{x\to -4} x^2 = 16.$

1) verify the limit

$\lim_{x\to -4} x^2 = (-4)^2 = 16 \checkmark$

2) $f(x)=x^2$, $a=-4$, $L=16$

3) $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2-16|<\varepsilon$ " $|x-(-4)|<\delta$

$|(x-4)(x+4)|<\varepsilon$ " $|x+4|<\delta$

keep

$|x-4|\,|x+4|<\varepsilon$ " $|x+4|<\delta$

Bound

for polynomial function, we want $\delta\le 1$

$|x+4|<\delta$

$|x+4|<1$

$-1<x+4<1$

subtract 8

$-1-8<x+4-8<1-8$

$-9<x-4<-7<9$

we had

$|x-4|\,|x+4|<9|x+4|<\varepsilon$

$-9<x-4<9$

$|x-4|<9$

$|x+4|<\frac{\varepsilon}{9}$

choose $\delta=\min\left\{1,\frac{\varepsilon}{9}\right\}$

Redo last example for $\delta \le 2$

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|x^2 - 16| < \varepsilon$ " $|x-(-4)| < \delta$

$|x-4||x+4| < \varepsilon$ " $|x+4| < \delta$

Bound Keep

for $\delta \le 2$

$|x+4| < 2$

$-2 < x+4 < 2$

Subtract 4

$-6 < x < -2$

Subtract 4

$-6-4 < x-4 < -2-4$

$-10 < x-4 < -6 < 10$

$-10 < x-4 < 10$

$|x-4| < 10$

we had

$|x-4||x+4| < 10|x+4| < \varepsilon$

$|x+4| < \frac{\varepsilon}{10}$

Choose $\delta = \min\left\{2, \frac{\varepsilon}{10}\right\}$

For $\varepsilon > 0$, find a $\delta > 0$ such that

$\lim_{x \to 3} x^3 = 27$ ✓

1) verify the limit

$\lim_{x \to 3} x^3 = 3^3 = 27$ ✓

2) $f(x) = x^3$, $a = 3$, $L = 27$

3)

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$A^3 - B^3$

$|x^3 - 27| < \varepsilon$ " $|x-3| < \delta$

$|(x-3)(x^2+3x+9)| < \varepsilon$ Keep

$|x-3||x^2+3x+9| < \varepsilon$

Bound

$f(x) = x^3$, Polynomial function $\Rightarrow$ δ no more than 1

$\delta \le 1$

$|x-3| < 1$

$-1 < x-3 < 1$

Add 3

$2 < x < 4$

If $\boxed{x < 4}$ $\Rightarrow$ $|x| < 4$

$x^2 < 16$

$3x < 12$

$x^2 + 3x + 9 < 16 + 12 + 9$

$|x^2 + 3x + 9| < 37$

So

$|x-3||x^2+3x+9| < 37|x-3| < \varepsilon$

and

$37|x-3| < \varepsilon$

$|x-3| < \frac{\varepsilon}{37}$

Choose $\delta = \min\left\{1, \frac{\varepsilon}{37}\right\}$

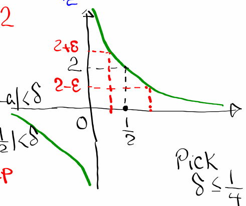
For $\varepsilon>0$, find a $\delta>0$ such that

$\lim_{x\to\frac12}\frac{1}{x}=2$ ✓

1) Verify the limit.

$\lim_{x\to 1/2}\frac{1}{x}=\frac{1}{1/2}=\boxed{2}$ ✓

2) $f(x)=\frac{1}{x}$, $a=\frac12$, $L=2$



3)

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\left|\frac{1}{x}-2\right|<\varepsilon$, $\underbrace{\left|x-\frac12\right|<\delta}_{\text{Keep}}$

$\left|\frac{1-2x}{x}\right|<\varepsilon$

Pick $\delta\le\frac14$

$\left|\frac{-2(x-\frac12)}{x}\right|<\varepsilon \Rightarrow \underbrace{\frac{|-2|}{|x|}}_{\text{Bound}}\underbrace{\left|x-\frac12\right|}_{\text{Keep}}<\varepsilon$

we need δ to be no more than $\frac14$

$\frac{2}{|x|}\left|x-\frac12\right|<\varepsilon$

$\left|x-\frac12\right|<\frac14$

$-\frac14<x-\frac12<\frac14$

$-\frac14+\frac12<x<\frac14+\frac12$

$\frac14<x<\frac34$

$4>\frac{1}{x}>\frac43$

$\frac43<\frac1x<4$

$-4<\frac1x<4$

$\left|\frac1x\right|<4$

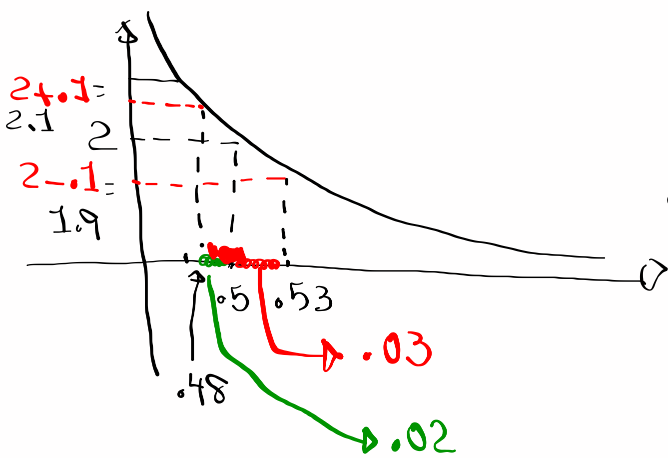
$2\left|\frac1x\right|<8$

Now $\frac{2}{|x|}\left|x-\frac12\right|<8\left|x-\frac12\right|<\varepsilon$

$\left|x-\frac12\right|<\frac{\varepsilon}{8}$ $\quad\delta=\min\left\{\frac14,\frac{\varepsilon}{8}\right\}$

If $\varepsilon=.1$

$\delta=\min\left\{\frac14,\frac{.1}{8}\right\}=\min\{.25,.0125\}\Rightarrow\boxed{\delta=.0125}$



Let's pick

$\varepsilon=.1$

$2.1=\frac{1}{x}$

$x=\frac{1}{2.1}\approx .48$

$1.9=\frac{1}{x}$

$x=\frac{1}{1.9}=.53$

Find a max. value for $\delta \Rightarrow .02$

.03 will not be given us $2\pm.1$

For $\varepsilon > 0$, find $\delta > 0$ such that

$\lim_{x \to 1} \frac{2}{x} = 2$ ✓

1) verify the limit

$\lim_{x \to 1} \frac{2}{x} = \frac{2}{1} = \boxed{2}$ ✓

2) $f(x) = \frac{2}{x}$, $a = 1$, $L = 2$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

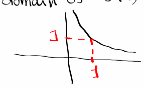
$\left| \frac{2}{x} - 2 \right| < \varepsilon$ 〃 $|x - 1| < \delta$

$\left| \frac{-2(x-1)}{x} \right| < \varepsilon$ 〃 $|x - 1| < \delta$

$\underbrace{\frac{2}{|x|}}_{\text{Bound}} \underbrace{|x - 1|}_{\text{keep}} < \varepsilon$ 〃 $|x - 1| < \delta$

We must have $\delta \le 1/2$

Be aware of domain of $f(x)$



$|x - 1| < .5$

$-.5 < x - 1 < .5$

$.5 < x < 1.5$

$2 > \frac{1}{x} > \frac{2}{3}$

$\frac{1}{|x|} < 2$ $\quad \frac{2}{|x|} < 4$

$\frac{2}{|x|} |x - 1| < 4|x - 1| < \varepsilon \Rightarrow |x - 1| < \frac{\varepsilon}{4}$

choose $\delta = \min\left\{\frac{1}{2}, \frac{\varepsilon}{4}\right\}$

Suppose $\varepsilon = \frac{1}{2}$

$\delta = \min\left\{\frac{1}{2}, \frac{1/2}{4}\right\} = \min\left\{1/2, \frac{1}{8}\right\} = .125$

$1.5 = \frac{1}{x} \Rightarrow x = \frac{1}{1.5} = \frac{2}{3}$

$.5 = \frac{1}{x} \Rightarrow x = 2$